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RECENT PROGRESS IN INFLATIONARY COSMOLOGY[1]

Andrei Linde

Department of Physics, Stanford University, Stanford CA 94305-4060, USA

ABSTRACT

We discuss two important modifications of inflationary paradigm. Until very recently we believed that inflation automatically leads to flatness of the universe, $\Omega = 1 \pm 10^{-4}$. We also thought that post-inflationary phase transitions in GUTs may occur only after thermalization, which made it very difficult to have baryogenesis in GUTs and to obtain superheavy topological defects after inflation. We will describe a very simple version of chaotic inflation which leads to a division of the universe into infinitely many open universes with all possible values of Ω from 1 to 0. We will show also that in many inflationary models quantum fluctuations of scalar and vector fields produced during reheating are much greater than they would be in a state of thermal equilibrium. This leads to cosmological phase transitions of a new type, which may result in an efficient GUT baryogenesis, in a copious production of topological defects and in a secondary stage of inflation after reheating.

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1 Inflation with $\Omega \neq 1$

One of the most robust predictions of inflationary cosmology is that the universe after inflation becomes extremely flat, which corresponds to $\Omega = 1$. Here $\Omega = \frac{\rho}{\rho_c}$, ρ_c being the energy density of a flat universe. There were many good reasons to believe that this prediction was quite generic. The only way to avoid this conclusion is to assume that the universe inflated only by about e^{60} times. Exact value of the number of e-folds N depends on details of the theory and may somewhat differ from 60. It is important, however, that in any particular theory inflation by extra 2 or 3 e-foldings would make the universe with $\Omega = 0.5$ or with $\Omega = 1.5$ almost exactly flat. Meanwhile, the typical number of e-foldings in chaotic inflation scenario in the theory $\frac{m^2}{2}\phi^2$ is not 60 but rather 10^{12}.

One can construct models where inflation leads to expansion of the universe by the factor e^{60}. However, in most of such models small number of e-foldings simultaneously implies that density perturbations are extremely large. It may be possible to overcome this obstacle by a specific choice of the effective potential. However, this would be only a partial solution. If the universe does not inflate long enough to become flat, then by the same token it does not inflate long enough to become homogeneous and isotropic. Thus, the main reason why it is difficult to construct inflationary models with $\Omega \neq 1$ is not the issue of fine tuning of the parameters of the models, which is necessary to obtain the universe inflating exactly e^{60} times, but the problem of obtaining a homogeneous universe after inflation.

Fortunately, it is possible to solve this problem, both for a closed universe (Linde 1992) and for an open one (Coleman and De Luccia, 1980, Gott 1982, Sasaki et al, 1993). The main idea is to use the well known fact that the region of space created in the process of a quantum tunneling tends to have a spherically symmetric shape, and homogeneous interior, if the tunneling process is suppressed strongly enough. Then such bubbles of a new phase tend to evolve (expand) in a spherically symmetric fashion. Thus, if one could associate the whole visible part of the universe with an interior of one such region, one would solve the homogeneity problem, and then all other problems will be solved by the subsequent relatively short stage of inflation.

For a closed universe the realization of this program is relatively straightforward (Linde, 1992, 1995). One should consider the process of quantum creation of a closed inflationary universe from "nothing." If the probability of such a process is exponentially suppressed (and this is indeed the case if inflation is possible only at the energy density much smaller than the Planck density (Linde, 1984, Vilenkin, 1984), then the universe created that way will be rather homogeneous from the very beginning.

The situation with an open universe is much more complicated. Indeed, an open universe is infinite, and it may seem impossible to create an infinite universe by a tunneling process. Fortunately, this is not the case: any bubble formed in the process of the false vacuum decay looks from inside like an infinite open universe (Coleman and De Luccia, 1980, Gott 1982, Sasaki *et al*, 1993). If this universe continues inflating inside the bubble (Gott 1982, Bucher *et al*, 1995) then we obtain an open inflationary universe.

There is an extensive investigation of the one-bubble open universe scenario, and many important results have been obtained, see e.g.

(Tanaka and Sasaki, 1994, Sasaki *et al*, 1995, Yamamoto *et al*, 1995, Bucher *et al*, 1995, Bucher and Turok, 1995, Hamazaki *et al*, 1995). However, for a long time it was not quite clear whether it is possible to realize this scenario in a natural way. It would be very nice to to obtain an open universe in a theory of just one scalar field, but in practice it is rather difficult to obtain a satisfactory model of this type. Typically one is forced either to introduce very complicated effective potentials, or consider theories with nonminimal kinetic terms for the inflaton field. This makes the models fine-tuned and complicated. It is very good to know that the models of such type in principle can be constructed, but it is also very tempting to find a more natural realization of the inflationary universe scenario which would give inflation with $\Omega < 1$.

Fortunately, this goal can be easily achieved if one considers models of two scalar fields (Linde, 1995, Linde and Mezhlumian, 1995, García–Bellido, 1995). One of these fields may be the standard inflaton field ϕ with a relatively small mass, another may be, e.g., the scalar field responsible for the symmetry breaking in GUTs. The presence of two scalar fields allows one to obtain the required bending of the inflaton potential by simply changing the definition of the inflaton field in the process of inflation. At the first stage the role of the inflaton is played by a heavy field with a steep barrier in its potential, while on the second stage the role of the inflaton is played by a light field, rolling in a flat direction "orthogonal" to the direction of quantum tunneling. This change of the direction of evolution in the space of scalar fields removes the naturalness constraints for the form of the potential, which are present in the case of one field.

Inflationary models of this type are quite simple, yet they have many interesting features. In these models the universe consists of infinitely many expanding bubbles immersed into exponentially expanding false vacuum state. Each of these bubbles inside looks like an open universe, but the values of Ω in these universes may take any value from 1 to 0. In some of these models the situation is even more complicated: Interior of each bubble looks like an infinite universe with an effective value of Ω slowly decreasing to $\Omega = 0$ at an exponentially large distance from the center of the bubble. We will call such universes quasiopen. Thus, rather unexpectedly, we are obtaining a large variety of interesting and previously unexplored possibilities.

Here we will describe an extremely simple model of two scalar fields, where the universe after inflation becomes open (or quasiopen, see below) in a very natural way (Linde, 1995, Linde and Mezhlumian, 1995).

Consider a model of two noninteracting scalar fields, ϕ and σ, with the effective potential

$$V(\phi,\sigma) = \frac{m^2}{2}\phi^2 + V(\sigma) \ . \tag{1}$$

Here ϕ is a weakly interacting inflaton field, and σ, for example, can be the field responsible for the symmetry breaking in GUTs. We will assume that $V(\sigma)$ has a local minimum at $\sigma = 0$, and a global minimum at $\sigma_0 \neq 0$, just as in the old inflationary theory. For definiteness, we will assume that this potential is given by $\frac{M^2}{2}\sigma^2 - \alpha M\sigma^3 + \frac{\lambda}{4}\sigma^4 + V(0)$, with $V(0) \sim \frac{M^4}{4\lambda}$, but it is not

essential; no fine tuning of the shape of this potential will be required.

Note that so far we did not make any unreasonable complications to the standard chaotic inflation scenario; at large ϕ inflation is driven by the field ϕ, and the GUT potential is necessary in the theory anyway. In order to obtain density perturbations of the necessary amplitude the mass m of the scalar field ϕ should be of the order of $10^{-6}M_{\rm P} \sim 10^{13}$ GeV (Linde, 1990).

Inflation begins at $V(\phi,\sigma) \sim M_{\rm P}^4$. At this stage fluctuations of both fields are very strong, and the universe enters the stage of self-reproduction, which finishes for the field ϕ only when it becomes smaller than $M_{\rm P}\sqrt{\frac{M_{\rm P}}{m}}$ and the energy density drops down to $mM_{\rm P}^3 \sim 10^{-6}M_{\rm P}^4$ (Linde, 1990). Quantum fluctuations of the field σ in some parts of the universe put it directly to the absolute minimum of $V(\sigma)$, but in some other parts the scalar field σ appears in the local minimum of $V(\sigma)$ at $\sigma = 0$. We will follow evolution of such domains. Since the energy density in such domains will be greater, their volume will grow with a greater speed, and therefore they will be especially important for us.

One may worry that all domains with $\sigma = 0$ will tunnel to the minimum of $V(\sigma)$ at the stage when the field ϕ was very large and quantum fluctuations of the both fields were large too. This may happen if the Hubble constant induced by the scalar field ϕ is much greater than the curvature of the potential $V(\sigma)$:

$$\frac{m\phi}{M_{\rm P}} \gtrsim M \ . \tag{2}$$

This decay can be easily suppressed if one introduces a small interaction $g^2\phi^2\sigma^2$ between these two fields, which stabilizes the state with $\sigma = 0$ at large ϕ. Another possibility is to add a nonminimal interaction with gravity of the form $-\frac{\xi}{2}R\phi^2$, which makes inflation impossible for $\phi > \frac{M_{\rm P}}{8\phi\xi}$. In this case the condition (fs1) will never be satisfied. However, there is a much simpler answer to this worry. If the effective potential of the field ϕ is so large that the field σ can easily jump to the true minimum of $V(\sigma)$, then the universe becomes divided into infinitely many domains with all possible values of σ distributed in the following way (Linde, 1990):

$$\frac{P(\sigma=0)}{P(\sigma=\sigma_0)} \sim \exp\left(\frac{3M_{\rm P}^4}{8V(\phi,0)} - \frac{3M_{\rm P}^4}{8V(\phi,\sigma)}\right) = \exp\left(\frac{3M_{\rm P}^4}{4(m^2\phi^2+2V(0))} - \frac{3M_{\rm P}^4}{4m^2\phi^2}\right) \ . \tag{3}$$

One can easily check that at the moment when the field ϕ decreases to $\frac{MM_{\rm P}}{m}$ and the condition (fs1) becomes violated, we will have

$$\frac{P(0)}{P(\sigma_0)} \sim \exp\left(-\frac{C}{\lambda}\right) \ , \tag{4}$$

where C is some constant, $C = O(1)$. After this moment the probability of the false vacuum decay typically becomes much smaller. Thus the fraction of space which survives in the false vacuum state $\sigma = 0$ until this time typically is very small, but finite (and calculable). It is important, that these rare domains with $\sigma = 0$ eventually will dominate the volume of the universe since if the probability of the false vacuum decay is small enough, the volume of the domains in the false vacuum will continue growing exponentially without end.

The main idea of our scenario can be explained as follows. Because the fields σ and ϕ do not interact with each other, and the dependence of the probability of tunneling on the vacuum energy at the GUT scale is negligibly small (Coleman and De Luccia, 1980), tunneling to the minimum of $V(\sigma)$ may occur with approximately equal probability at all sufficiently small values of the field ϕ (see, however, below). The parameters of the bubbles of the field σ are determined by the mass scale M corresponding to the effective potential $V(\sigma)$. This mass scale in our model is much greater than m. Thus the duration of tunneling in the Euclidean "time" is much smaller than m^{-1}. Therefore the field ϕ practically does not change its value during the tunneling. If the probability of decay at a given ϕ is small enough, then it does not destroy the whole vacuum state $\sigma = 0$; the bubbles of the new phase are produced all the way when the field ϕ rolls down to $\phi = 0$. In this process the universe becomes filled with (nonoverlapping) bubbles immersed in the false vacuum state with $\sigma = 0$. Interior of each of these bubbles represents an open universe. However, these bubbles contain *different* values of the field ϕ, depending on the value of this field at the moment when the bubble formation occurred. If the field ϕ inside a bubble is smaller than $3M_{\rm P}$, then the universe inside this bubble will have a vanishingly small Ω, at the age 10^{10} years after the end of inflation it will be practically empty, and life of our type would not exist there. If the field ϕ is much greater than $3M_{\rm P}$, the universe inside the bubble will be almost exactly flat, $\Omega = 1$, as in the simplest version of the chaotic inflation scenario. It is important, however, that *in an eternally existing self-reproducing universe there will be infinitely many universes containing any particular value of* Ω*, from* $\Omega = 0$ *to* $\Omega = 1$, and one does not need any fine tuning of the effective potential to obtain a universe with, say, $0.2 < \Omega < 0.3$

Of course, one can argue that we did not solve the problem of fine tuning, we just transformed it into the fact that only a very small percentage of all universes will have $0.2 < \Omega < 0.3$. However, first of all, we achieved our goal in a very simple theory, which does not require any artificial potential bending and nonminimal kinetic terms. Then, there may be some reasons why it is preferable for us to live in a universe with a small (but not vanishingly small) Ω.

The simplest way to approach this problem is to find how the probability for the bubble production depends on ϕ. As we already pointed out, for small ϕ this dependence is not very strong. On the other hand, at large ϕ the probability rapidly grows and becomes quite large at $\phi > \frac{MM_{\rm P}}{m}$. This may suggest that the bubble production typically occurs at $\phi > \frac{MM_{\rm P}}{m}$, and then for $\frac{M}{m} \gg 3$ we typically obtain flat universes, $\Omega = 1$. This is another manifestation of the problem of premature decay of the state $\sigma = 0$ which we discussed above. Moreover, even if the probability to produce the universes with different ϕ were entirely ϕ-independent, one could argue that the main volume of the habitable parts of the universe is contained in the bubbles with $\Omega = 1$, since the interior of each such bubble inflated longer. Indeed, the total volume of each bubble created in a state with the field ϕ during inflation in our model grows by the factor of $\exp \frac{6\pi\phi^2}{M_{\rm P}^2}$ (Linde, 1990). It seems clear that the bubbles with greater ϕ will give the largest contribution to the total volume of the universe after inflation. This would be the simplest argument in favor of the standard prediction $\Omega = 1$ even in our class of models.

However, there exist several ways of resolving this problem: involving coupling $g^2\phi^2\sigma^2$, which stabilizes the state $\sigma = 0$ at large ϕ, or adding nonminimal interaction with gravity of the form

$-\frac{\xi}{2}R\phi^2$. In either way one can easily suppress production of the universes with $\Omega = 1$. Then the maximum of probability will correspond to some value $\Omega < 1$, which can be made equal to any given number from 1 to 0 by changing the parameters g^2 and ξ.

For example, let us add to the Lagrangian the term $-\frac{\xi}{2}R\phi^2$. This term makes inflation impossible for $\phi > \phi_c = \frac{M_{\rm P}}{\sqrt{8\pi\xi}}$. If initial value of the field ϕ is much smaller than ϕ_c, the size of the universe during inflation grows $\exp\frac{2\pi\phi^2}{M_{\rm P}^2}$ times, and the volume grows $\exp\frac{6\pi\phi^2}{M_{\rm P}^2}$ times, as in the theory $\frac{m^2}{2}\phi^2$ with $\xi = 0$. For initial ϕ approaching ϕ_c these expressions somewhat change, but in order to get a very rough estimate of the increase of the size of the universe in this model (which is sufficient to get an illustration of our main idea) one can still use the old expression $\exp\frac{2\pi\phi^2}{M_{\rm P}^2}$. This expression reaches its maximum near $\phi = \phi_c$, at which point the effective gravitational constant becomes infinitely large and inflationary regime ceases to exist (Futamase, 1989, García–Bellido and Linde, 1995). Thus, one may argue that in this case the main part of the volume of the universe will appear from the bubbles with initial value of the field ϕ close to ϕ_c. For $\xi \ll 4.4 \times 10^{-3}$ one has $\phi_c \gg 3M_{\rm P}$. In this case one would have typical universes expanding much more than e^{60} times, and therefore $\Omega \approx 1$. For $\xi \gg 4.4 \times 10^{-3}$ one has $\phi_c \ll 3M_{\rm P}$, and therefore one would have $\Omega \ll 1$ in all inflationary bubbles. It is clear that by choosing particular values of the constant ξ in the range of $\xi \sim 4.4 \times 10^{-3}$ one can obtain the distribution of the universes with the maximum of the distribution concentrated near any desirable value of $\Omega < 1$. Note that the position of the peak of the distribution is very sensitive to the value of ξ: to have the peak concentrated in the region $0.2 < \Omega < 0.3$ one would have to fix ξ (i.e. ϕ_c) with an accuracy of few percent. Thus, in this approach to the calculation of probabilities to live in a universe with a given value of Ω we still have the problem of fine tuning.

However, calculation of probabilities in the context of the theory of a self-reproducing universe is a very ambiguous process, and it is even not quite clear that this process makes any sense at all. For example, we may formulate the problem in a different way. Consider a domain of the false vacuum with $\sigma = 0$ and $\phi = \phi_1$. After some evolution it produces one or many bubbles with $\sigma = \sigma_0$ and the field ϕ which after some time becomes equal to ϕ_2. One may argue that the most efficient way this process may go is the way which in the end produces the greater volume. Indeed, for the inhabitants of a bubble it does not matter how much time did it take for this process to occur. The total number of observers produced by this process will depend on the total volume of the universe at the hypersurface of a given density, i.e. on the hypersurface of a given ϕ. If the domain instantaneously tunnels to the state σ_0 and ϕ_1, and then the field ϕ in this domain slowly rolls from ϕ_1 to ϕ_2, then the volume of this domain grows $\exp\Big(\frac{2\pi}{M_{\rm P}^2}(\phi_1^2 - \phi_2^2)\Big)$ times (Linde, 1990). Meanwhile, if the tunneling takes a long time, then the field ϕ rolls down extremely slowly being in the false vacuum state with $\sigma = 0$. In this state the universe expands much faster than in the state with $\sigma = \sigma_0$. Since it expands much faster, and it takes the field much longer to roll from ϕ_1 to ϕ_2, the trajectories of this kind bring us much greater volume. This may serve as an argument that most of the volume is produced by the bubbles created at a very small ϕ, which leads to the universes with very small Ω.

One may use another set of considerations, studying all trajectories beginning at ϕ_1, t_1 and

ending at ϕ_2, t_2. This will bring us another answer, or, to be more precise, another set of answers, which will depend on the choice of the time parametrization (Linde *et al*, 1994). Still another answer will be obtained by the method recently proposed by Vilenkin, who suggested to introduce a particular cutoff procedure which partially eliminates dependence of the final answer on the time parametrization (Vilenkin, 1995, Winitzki and Vilenkin, 1995)). However, there exists a wide class of cutoff procedures which have similar properties, but give exponentially different results (Linde and Mezhlumian, 1995a)

There is a very deep reason why the calculation of the probability to obtain a universe with a given Ω is so ambiguous. We have discussed this reason in Sect. 3.1 in general terms; let us see how the situation looks in application to the open universe scenario. For those who lives inside a bubble there is be no way to say at which stage (at which time from the point of view of an external observer) this bubble was produced. Therefore one should compare *all* of these bubbles produced at all possible times. The self-reproducing universe should exist for indefinitely long time, and therefore it should contain infinitely many bubbles with all possible values of Ω. Comparing infinities is a very ambiguous task, which gives results depending on the procedure of comparison. For example, one can consider an infinitely large box of apples and an infinitely large box of oranges. One may pick up one apple and one orange, then one apple and one orange, over and over again, and conclude that there is an equal number of apples and oranges. However, one may also pick up one apple and two oranges, and then one apple and two oranges again, and conclude that there is twice as many oranges as apples. The same situation happens when one tries to compare the number of bubbles with different values of Ω. If we would know how to solve the problem of measure in quantum cosmology, perhaps we would be able to obtain something similar to an open universe in the trivial $\lambda\phi^4$ theory without any first order phase transitions (Linde *et al* 1995, 1995a). In the meantime, it is already encouraging that in our scenario there are infinitely many inflationary universes with all possible value of $\Omega < 1$. We can hardly live in the empty bubbles with $\Omega = 0$. As for the choice between the bubbles with different nonvanishing values of $\Omega < 1$, it is quite possible that eventually we will find out an unambiguous way of predicting the most probable value of Ω, and we are going to continue our work in this direction. However, as we already discussed in the previous section, it might also happen that this question is as meaningless as the question whether it is more probable to be born as a Chinese rather than as an Italian. It is quite conceivable that the only way to find out in which of the bubbles do we live is to make observations.

Some words of caution are in order here. The bubbles produced in our simple model are not *exactly* open universes. Indeed, in the one-field models the time of reheating (and the temperature of the universe after the reheating) was exactly synchronized with the value of the scalar field inside the bubble. In our case the situation is very similar, but not exactly. Suppose that the Hubble constant induced by $V(0)$ is much greater than the Hubble constant related to the energy density of the scalar field ϕ. Then the speed of rolling of the scalar field ϕ sharply increases inside the bubble. Thus, in our case the field σ synchronizes the motion of the field ϕ, and then the hypersurface of a constant field ϕ determines the hypersurface of a constant temperature. In the models where the rolling of the field ϕ can occur only inside the bubble (we will discuss such a model shortly) the synchronization is precise, and everything goes as in the one-field models.

However, in our simple model the scalar field ϕ moves down outside the bubble as well, even though it does it very slowly. Thus, synchronization of motion of the fields σ and ϕ is not precise; hypersurface of a constant σ ceases to be a hypersurface of a constant density. For example, suppose that the field ϕ has taken some value ϕ_0 near the bubble wall when the bubble was just formed. Then the bubble expands, and during this time the field ϕ outside the wall decreases, as $\exp\Bigl(-\frac{m^2 t}{3H_1}\Bigr)$, where $H_1 \approx H(\phi = \sigma = 0)$ is the Hubble constant at the first stage of inflation, $H_1 \approx \sqrt{\frac{8\pi V(0)}{3M_{\rm P}^2}}$. At the moment when the bubble expands e^{60} times, the field ϕ in the region just reached by the bubble wall decreases to $\phi_o \exp\Bigl(-\frac{20m^2}{H_1^2}\Bigr)$ from its original value ϕ_0. the universe inside the bubble is a homogeneous open universe only if this change is negligibly small. This may not be a real problem. Indeed, let us assume that $V(0) = \tilde{M}^4$, where $\tilde{M} = 10^{17}$ GeV. (Typically the energy density scale $\tilde{M}$ is related to the particle mass as follows: $\tilde{M} \sim \lambda^{-1/4} M$.) In this case $H_1 = 1.7 \times 10^{15}$ GeV, and for $m = 10^{13}$ GeV one obtains $\frac{20m^2}{H_1^2} \sim 10^{-4}$. In such a case a typical degree of distortion of the picture of a homogeneous open universe is very small.

Still this issue requires careful investigation. When the bubble wall continues expanding even further, the scalar field outside of it eventually drops down to zero. Then there will be no new matter created near the wall. Instead of infinitely large homogeneous open universes we are obtaining spherically symmetric islands of a size much greater than the size of the observable part of our universe. We do not know whether this unusual picture is an advantage or a disadvantage of our model. Is it possible to consider different parts of the same exponentially large island as domains of different "effective" Ω? Can we attribute some part of the dipole anisotropy of the microwave background radiation to the possibility that we live somewhere outside of the center of such island? In any case, as we already mentioned, in the limit $m^2 \ll H_1^2$ we do not expect that the small deviations of the geometry of space inside the bubble from the geometry of an open universe can do much harm to our model.

Our model admits many generalizations, and details of the scenario which we just discussed depend on the values of parameters. Let us forget for a moment about all complicated processes which occur when the field ϕ is rolling down to $\phi = 0$, since this part of the picture depends on the validity of our ideas about initial conditions. For example, there may be no self-reproduction of inflationary domains with large ϕ if one considers an effective potential of the field ϕ which is very curved at large ϕ. However, there will be self-reproduction of the universe in a state $\phi = \sigma = 0$, as in the old inflation scenario. Then the main portion of the volume of the universe will be determined by the processes which occur when the fields ϕ and σ stay at the local minimum of the effective potential, $\phi = \sigma = 0$. For definiteness we will assume here that $V(0) = \tilde{M}^4$, where $\tilde{M}$ is the stringy scale, $\tilde{M} \sim 10^{17} - 10^{18}$ GeV. Then the Hubble constant $H_1 = \sqrt{\frac{8\pi V(0)}{3M_{\rm P}^2}} \sim \sqrt{\frac{8\pi}{3}}\frac{\tilde{M}^2}{M_{\rm P}}$ created by the energy density $V(0)$ is much greater than $m \sim 10^{13}$ GeV. In such a case the scalar field ϕ will not stay exactly at $\phi = 0$. It will be relatively homogeneous on the horizon scale H_1^{-1}, but otherwise it will be chaotically distributed with the dispersion $\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 m^2}$ (Linde, 1990). This means that the field ϕ inside each of the bubbles produced by the decay of the false vacuum

can take any value ϕ with the probability

$$P \sim \exp\left(-\frac{\phi^2}{2\langle\phi^2\rangle}\right) \sim \exp\left(-\frac{3m^2\phi^2 M_{\rm P}^4}{16\tilde{M}^8}\right) \ . \tag{5}$$

One can check that for $\tilde{M} \sim 4.3 \times 10^{17}$ GeV the typical value of the field ϕ inside the bubbles will be $\sim 3 \times 10^{19}$ GeV. Thus, for $\tilde{M} > 4.3 \times 10^{17}$ GeV most of the universes produced during the vacuum decay will be flat, for $\tilde{M} < 4.3 \times 10^{17}$ GeV most of them will be open. It is interesting that in this version of our model the percentage of open universes is determined by the stringy scale (or by the GUT scale). However, since the process of bubble production in this scenario goes without end, the total number of universes with any particular value of $\Omega < 1$ will be infinitely large for any value of $\tilde{M}$. Thus this model shows us is the simplest way to resurrect some of the ideas of the old inflationary theory with the help of chaotic inflation, and simultaneously to obtain inflationary universe with $\Omega < 1$.

Note that this version of our model will not suffer for the problem of incomplete synchronization. Indeed, the average value of the field ϕ in the false vacuum outside the bubble will remain constant until the bubble triggers its decrease. However, this model, just as its previous version, may suffer from another problem. The Hubble constant H_1 before the tunneling in this model was much greater than the Hubble constant H_2 at the beginning of the second stage of inflation. Therefore the fluctuations of the scalar field before the tunneling were very large, $\delta\phi \sim \frac{H_1}{2\pi}$, much greater than the fluctuations generated after the tunneling, $\delta\phi \sim \frac{H_2}{2\pi}$. This may lead to very large density perturbations on the scale comparable to the size of the bubble. For the models with $\Omega = 1$ this effect would not cause any problems since such perturbations would be far away over the present particle horizon, but for small Ω this may lead to unacceptable anisotropy of the microwave background radiation.

Fortunately, this may not be a real difficulty. A possible solution is very similar to the bubble symmetrization described in the previous section.

Indeed, let us consider more carefully how the long wave perturbations produced outside the bubble may penetrate into it. At the moment when the bubble is formed, it has a size smaller than H_1^{-1} (Coleman and De Luccia, 1980). Then the bubble walls begin moving with the speed gradually approaching the speed of light. At this stage the comoving size of the bubble (from the point of view of the original coordinate system in the false vacuum) grows like

$$r(t) = \int_0^t dte^{-H_1 t} = H_1^{-1}(1 - e^{-H_1 t}) \ . \tag{6}$$

During this time the fluctuations of the scalar field ϕ of the amplitude $\frac{H_1}{2\pi}$ and of the wavelength H_1^{-1}, which previously were outside the bubble, gradually become covered by it. When these perturbations are outside the bubble, inflation with the Hubble constant H_1 prevents them from oscillating and moving. However, once these perturbations penetrate inside the bubble, their amplitude becomes decreasing (Mukhanov and Zelnikov, 1991). Indeed, since the wavelength of the perturbations is $\sim H_1^{-1} \ll H_2^{-1} \ll m^{-1}$, these perturbations move inside the bubbles as relativistic particles, their wavelength grow as $a(t)$, and their amplitude decreases just like

an amplitude of electromagnetic field, $\delta\phi \sim a^{-1}(t)$, where a is the scale factor of the universe inside a bubble (Mukhanov and Zelnikov, 1991). This process continues until the wavelength of each perturbation reaches H_2^{-1} (already at the second stage of inflation). During this time the wavelength grows $\frac{H_1}{H_2}$ times, and the amplitude decreases $\frac{H_2}{H_1}$ times, to become the standard amplitude of perturbations produced at the second stage of inflation: $\frac{H_2}{H_1}\frac{H_1}{2\pi} = \frac{H_2}{2\pi}$.

In fact, one may argue that this computation was too naive, and that these perturbations should be neglected altogether. Typically we treat long wave perturbations in inflationary universe like classical wave for the reason that the waves with the wavelength much greater than the horizon can be interpreted as states with extremely large occupation numbers (Linde, 1990). However, when the new born perturbations (i.e. fluctuations which did not acquire an exponentially large wavelength yet) enter the bubble (i.e. under the horizon), they effectively return to the realm of quantum fluctuations again. Then one may argue that one should simply forget about the waves with the wavelengths small enough to fit into the bubble, and consider perturbations created at the second stage of inflation not as a result of stretching of these waves, but as a new process of creation of perturbations of an amplitude $\frac{H_2}{2\pi}$.

One may worry that perturbations which had wavelengths somewhat greater than H_1^{-1} at the moment of the bubble formation cannot completely penetrate into the bubble. If, for example, the field ϕ differs from some constant by $+\frac{H_1}{2\pi}$ at the distance H_1^{-1} to the left of the bubble at the moment of its formation, and by $-\frac{H_1}{2\pi}$ at the distance H_1^{-1} to the right of the bubble, then this difference remains frozen independently of all processes inside the bubble. This may suggest that there is some unavoidable asymmetry of the distribution of the field inside the bubble. However, the field inside the bubble will not be distributed like a straight line slowly rising from $-\frac{H_1}{2\pi}$ to $+\frac{H_1}{2\pi}$. Inside the bubble the field will be almost homogeneous; the inhomogeneity $\delta\phi \sim -\frac{H_1}{2\pi}$ will be concentrated only in a small vicinity near the bubble wall.

Finally we should verify that this scenario leads to bubbles which are symmetric enough. Fortunately, here we do not have any problems. One can easily check that for our model with $m \sim 10^{13}$ GeV and $\tilde{M} \sim \lambda^{-1/4} M > 10^{17} GeV$ perturbations of metric induced by the wall perturbations are small even for not very small values of the coupling constant λ (Linde and Mezhlumian, 1995, García–Bellido, 1995).

The arguments presented above should be confirmed by a more detailed investigation of the vacuum structure inside the expanding bubble in our scenario. If, as we hope, the result of the investigation will be positive, we will have an extremely simple model of an open inflationary universe. In the meantime, it would be nice to have a model where we do not have any problems at all with synchronization and with large fluctuations on the scalar field in the false vacuum.

The simplest model of this kind is a version of the hybrid inflation scenario (Linde, 1991, 1994), which is a slight generalization (and a simplification) of our previous model (f3):

$$V(\phi,\sigma) = \frac{g^2}{2}\phi^2\sigma^2 + V(\sigma) \ . \tag{7}$$

We eliminated the massive term of the field ϕ and added explicitly the interaction $\frac{g^2}{2}\phi^2\sigma^2$, which,

as we have mentioned already, can be useful (though not necessary) for stabilization of the state $\sigma = 0$ at large ϕ. Note that in this model the line $\sigma = 0$ is a flat direction in the (ϕ, σ) plane. At large ϕ the only minimum of the effective potential with respect to σ is at the line $\sigma = 0$. To give a particular example, one can take $V(\sigma) = \frac{M^2}{2}\sigma^2 - \alpha M\sigma^3 + \frac{\lambda}{4}\sigma^4 + V_0$. Here V_0 is a constant which is added to ensure that $V(\phi,\sigma) = 0$ at the absolute minimum of $V(\phi,\sigma)$. In this case the minimum of the potential $V(\phi,\sigma)$ at $\sigma \neq 0$ is deeper than the minimum at $\sigma = 0$ only for $\phi < \phi_c$, where $\phi_c = \frac{M}{g}\sqrt{\frac{2\alpha^2}{\lambda} - 1}$. This minimum for $\phi = \phi_c$ appears at $\sigma = \sigma_c = \frac{2\alpha M}{\lambda}$.

The bubble formation becomes possible only for $\phi < \phi_c$. After the tunneling the field ϕ acquires an effective mass $m = g\sigma$ and begins to move towards $\phi = 0$, which provides the mechanism for the second stage of inflation inside the bubble. In this scenario evolution of the scalar field ϕ is exactly synchronized with the evolution of the field σ, and the universe inside the bubble appears to be open.

Effective mass of the field ϕ at the minimum of $V(\phi,\sigma)$ with $\phi = \phi_c$, $\sigma = \sigma_c = \frac{2\alpha M}{\lambda}$ is $m = g\sigma_c = \frac{2g\alpha M}{\lambda}$. With a decrease of the field ϕ its effective mass at the minimum of $V(\phi,\sigma)$ will grow, but not significantly. For simplicity, we will consider the case $\lambda = \alpha^2$. In this case it can be shown that $V(0) = 2.77\,\frac{M^4}{\lambda}$, and the Hubble constant before the phase transition is given by $4.8\,\frac{M^2}{\sqrt{\lambda}M_{\rm P}}$. The effective mass m after the phase transition is equal to $\frac{2gM}{\sqrt{\lambda}}$ at $\phi = \phi_c$, and then it grows by only 25% when the field ϕ changes all the way down from ϕ_c to $\phi = 0$.

The bubble formation becomes possible only for $\phi < \phi_c$. If it happens in the interval $4M_{\rm P} > \phi > 3M_{\rm P}$, we obtain a flat universe. If it happens at $\phi < 3M_{\rm P}$, we obtain an open universe. Depending on the initial value of the field ϕ, we can obtain all possible values of Ω, from $\Omega = 1$ to $\Omega = 0$. The value of the Hubble constant at the minimum with $\sigma \neq 0$ at $\phi = 3M_{\rm P}$ in our model does not differ much from the value of the Hubble constant before the bubble formation. Therefore we do not expect any specific problems with the large scale density perturbations in this model. Note also that the probability of tunneling at large ϕ is very small since the depth of the minimum at $\phi \sim \phi_c$, $\sigma \sim \sigma_c$ does not differ much from the depth of the minimum at $\sigma = 0$, and there is no tunneling at all for $\phi > \phi_c$. Therefore the number of flat universes produced by this mechanism will be strongly suppressed as compared with the number of open universes, the degree of this suppression being very sensitive to the value of ϕ_c. Meanwhile, life of our type is impossible in empty universes with $\Omega \ll 1$. This may provide us with a tentative explanation of the small value of Ω in the context of our model.

Another model of inflation with $\Omega < 1$ is the based on a certain modification of the "natural inflation" scenario (Freese *et al*, 1990). The main idea is to take the effective potential of the "natural inflation" model, which looks like a tilted Mexican hat, and make a deep hole in its center at $\phi = 0$ (Linde and Mezhlumian, 1995). In the beginning inflation occurs near $\phi = 0$, but then the bubbles with $\phi \neq 0$ appear. Depending on the phase of the complex scalar field ϕ inside the bubble, the next stage of inflation, which occurs just as in the old version of the "natural inflation" scenario, leads to formation of the universes with all possible values of Ω. Thus, there exist several simple inflationary models which lead to the picture of the universe consisting of many bubbles with different values of Ω. Therefore instead of insisting that inflation leads to

$\Omega = 1$ or estimating the probability to live in a bubble with a given value of Ω we should ask astronomers to measure it.

2 Nonthermal Phase Transitions after Inflation

The theory of reheating is one of the most important parts of inflationary cosmology. Elementary theory of this process was developed many years ago by Dolgov and Linde (1982) and by Abbott *et al* (1982). Some important steps toward a complete theory have been made in (Dolgov and Kirilova, 1990, Traschen and Brandenberger, 1990). However, the real progress in understanding of this process was achieved only recently when the new theory of reheating was developed. According to this theory (Kofman *et al*, 1994), reheating typically consists of three different stages. At the first stage, a classical oscillating scalar field ϕ (the inflaton field) decays into massive bosons due to parametric resonance. In many models the resonance is very broad, and the process occurs extremely rapidly. To distinguish this stage of explosive reheating from the stage of particle decay and thermalization, we called it *preheating*. Bosons produced at that stage are far away from thermal equilibrium and have enormously large occupation numbers. The second stage is the decay of previously produced particles. This stage typically can be described by the elementary theory developed by Dolgov and Linde (1982) and by Abbott *et al* (1982). However, these methods should be applied not to the decay of the original homogeneous inflaton field, but to the decay of particles produced at the stage of preheating. This changes many features of the process including the final value of the reheating temperature. The third stage of reheating is thermalization.

Different aspects of the theory of explosive reheating have been studied by many authors (Shtanov *et al*, 1995, Boyanovsky *et al*, 1995, Yoshimura, 1995, Kaiser, 1995, Fujisaki *et al*, 1995). In our presentation we will follow the original approach of Kofman *et al* (1994), where the theory of reheating was investigated with an account taken both of the expansion of the universe and of the backreaction of created particles. The results reported here have been obtained by Kofman *et al* (1995, 1996).

One should note that there exist such models where this first stage of reheating is absent; e.g, there is no parametric resonance in the theories where the field ϕ decays into fermions. However, in the theories where preheating is possible one may expect many unusual phenomena. One of the most interesting effects is the possibility of specific non-thermal post-inflationary phase transitions which occur after preheating. As we will see, these phase transitions in certain cases can be much more pronounced that the standard high temperature cosmological phase transitions. They may lead to copious production of topological defects and to a secondary stage of inflation after reheating.

Let us first remember the theory of phase transitions in theories with spontaneous symmetry breaking (Kirzhnits, 1972, Kirzhnits. and Linde, 1972, Weinberg, 1974, Dolan and Jackiw, 1974, Kirzhnits and Linde, 1974, 1976). We will consider first the theory of scalar fields ϕ and χ with

the effective potential

$$V(\phi,\chi) = \frac{\lambda}{4}(\phi^2-\phi_0^2)^2 + \frac{1}{2}g^2\phi^2\chi^2 \ . \tag{8}$$

Here $\lambda, g \ll 1$ are coupling constants. $V(\phi,\chi)$ has a minimum at $\phi = \phi_0$, $\chi = 0$ and a maximum at $\phi = \chi = 0$ with the curvature $V_{\phi\phi} = -m^2 = -\lambda\phi_0^2$. This effective potential acquires corrections due to quantum (or thermal) fluctuations of the scalar fields (Weinberg, 1974, Dolan and Jackiw, 1974, Kirzhnits and Linde, 1974), $\Delta V = \frac{3}{2}\lambda\langle(\delta\phi^2)\rangle\phi^2 + \frac{g^2}{2}\langle(\delta\chi)^2\rangle\phi^2 + \frac{g^2}{2}\langle(\delta\phi)^2\rangle\chi^2 + ...$, where the quantum field operator is decomposed as $\hat\phi = \phi + \delta\phi$ with $\phi \equiv \langle\hat\phi\rangle$, and we have written only leading terms depending on ϕ and $\chi \equiv \langle\hat\chi\rangle$. In the large temperature limit $\langle(\delta\phi)^2\rangle = \langle(\delta\chi)^2\rangle = \frac{T^2}{12}$. The effective mass squared of the field ϕ

$$m^2_{\phi,eff} = -m^2 + 3\lambda\phi^2 + 3\lambda\langle(\delta\phi)^2\rangle + g^2\langle(\delta\chi)^2\rangle \tag{9}$$

becomes positive and symmetry is restored (i.e. $\phi = 0$ becomes the stable equilibrium point) for $T > T_c$, where $T_c^2 = \frac{12m^2}{3\lambda+g^2} \gg m^2$. At this temperature the energy density of the gas of ultrarelativistic particles is given by $\rho = N(T_c)\frac{\pi^2}{30}T_c^4 = \frac{24\, m^4 N(T_c)\pi^2}{5\,(3\lambda+g^2)^2}$. Here $N(T)$ is the effective number of degrees of freedom at large temperature, which in realistic situations may vary from 10^2 to 10^3. Note that for $g^4 < \frac{96N\pi^2}{5}\lambda$ this energy density is greater than the vacuum energy density $V(0) = \frac{m^4}{4\lambda}$. Meanwhile, for $g^4 \gtrsim \lambda$ radiative corrections are important, they lead to creation of a local minimum of $V(\phi,\chi)$, and the phase transition occurs from a strongly supercooled state (Kirzhnits and Linde, 1976). That is why the first models of inflation required supercooling at the moment of the phase transition.

An exception from this rule is given by supersymmetric theories, where one may have $g^4 \gg \lambda$ and still have a potential which is flat near the origin due to cancellation of quantum corrections of bosons and fermions (Lyth and Stewart, 1995). In such cases thermal energy becomes smaller than the vacuum energy at $T < T_0$, where $T_0^4 = \frac{15}{2N\pi^2}m^2\phi_0^2$. Then one may even have a short stage of inflation which begins at $T \sim T_0$ and ends at $T = T_c$. During this time the universe may inflate by the factor

$$\frac{a_c}{a_0} = \frac{T_0}{T_c} \sim 10^{-1}\Big(\frac{g^4}{\lambda}\Big)^{1/4} \approx 10^{-1} g\sqrt{\frac{\phi_0}{m}}. \tag{10}$$

In supersymmetric theories with flat directions Φ it may be more natural to consider potentials of the so-called "flaton" fields Φ without the term $\frac{\lambda}{4}\Phi^4$ (Lyth and Stewart, 1995):

$$V(\Phi,\chi) = -\frac{m^2\Phi^2}{2} + \frac{\lambda_1\Phi^6}{6M_p^2} + \frac{m^2\Phi_0^2}{3} + \frac{1}{2}g^2\Phi^2\chi^2 \ , \tag{11}$$

where $\Phi_0 = \lambda_1^{-1/4}\sqrt{mM_p}$ corresponds to the minimum of this potential. The critical temperature in this theory for $\lambda_1\Phi_0^2 \ll g^2M_p^2$ is the same as in the theory (fp1) for $\lambda \ll g^2$, and expansion of the universe during thermal inflation is given by $10^{-1}g\sqrt{\Phi_0/m}$, as in eq. (fp5a). Existence of this short additional stage of "thermal inflation" is a very interesting effect, which may be very useful. In particular, it may provide a solution to the Polonyi field problem (Lyth and Stewart, 1995).

The theory of cosmological phase transitions is an important part of the theory of the evolution of the universe, and during the last twenty years it was investigated in a very detailed way. However, typically it was assumed that the phase transitions occur in the state of thermal equilibrium. Now we are going to show that similar phase transitions may occur even much more efficiently prior to thermalization, due to the anomalously large expectation values $\langle(\delta\phi)^2\rangle$ and $\langle(\delta\chi)^2\rangle$ produced during preheating.

We will first consider the model (8) without the scalar field χ and with the amplitude of spontaneous symmetry breaking $\phi_0 \ll M_{\rm P}$. In this model inflation occurs during the slow rolling of the scalar field ϕ from its very large values until it becomes of the order $M_{\rm P}$. Then it oscillates with the initial amplitude $\phi \sim 10^{-1}M_p$ and initial frequency $\sim 10^{-1}\sqrt{\lambda}M_{\rm P}$. Within a few dozen oscillations it transfers most of its energy $\sim \frac{\lambda}{4}10^{-4}M_{\rm P}^4$ to its long-wave fluctuations $\langle(\delta\phi)^2\rangle$ in the regime of broad parametric resonance (Kofman *et al*, 1994).

The crucial observation is the following. Suppose that the initial energy density of oscillations $\sim \frac{\lambda}{4}10^{-4}M_{\rm P}^4$ were instantaneously transferred to thermal energy density $\sim 10^2T^4$. This would give the reheating temperature $T_r \sim 2\times 10^{-2}\lambda^{1/4}M_{\rm P}$, and the scalar field fluctuations $\langle(\delta\phi)^2\rangle \sim T_r^2/12 \sim 3\times 10^{-5}\sqrt{\lambda}M_{\rm P}^2$. Meanwhile particles created during preheating have much smaller energy $\sim 10^{-1}\sqrt{\lambda}M_{\rm P}$. Therefore if the same energy density $\frac{\lambda}{4}10^{-4}M_{\rm P}^4$ is instantaneously transferred to low-energy particles created during preheating, their number, and, correspondingly, the amplitude of fluctuations, will be much greater, $\langle(\delta\phi)^2\rangle \sim C^2M_{\rm P}^2$, where $C^2 \sim 10^{-2} - 10^{-3}$ (Kofman *et al*, 1994, 1996). Thermal fluctuations would lead to symmetry restoration in our model only for $\phi_0 \lesssim T_r \sim 10^{-2}\lambda^{1/4}M_{\rm P} \sim 10^{14}$ GeV for the realistic value $\lambda \sim 10^{-13}$ (Linde, 1990). Meanwhile, according to eq. (9), the nonthermalized fluctuations $\langle(\delta\phi)^2\rangle \sim M_{\rm P}^2$ may lead to symmetry restoration even if the symmetry breaking parameter ϕ_0 is as large as $10^{-1}M_{\rm P}$. Thus, the nonthermal symmetry restoration may occur even in those theories where the symmetry restoration due to high temperature effects would be impossible (Kofman *et al*, 1995). (Recently a similar conclusion was reached also by Tkachev (1995). However, his investigation was based on an oversimplified picture of reheating, and his estimates differ considerably from our results.)

In reality thermalization takes a very long time, which is inversely proportional to coupling constants. This dilutes the energy density, and the reheating temperature becomes many orders of magnitude smaller than 10^{14} GeV (Linde, 1990). Therefore post-inflationary thermal effects typically cannot restore symmetry on the GUT scale. Preheating is not instantaneous as well, and therefore the fluctuations produced at that stage are smaller than $C^2M_{\rm P}^2$, but only logarithmically: $\langle(\delta\phi)^2\rangle \sim C^2M_{\rm P}^2\ln^{-2}\frac{1}{\lambda}$ (Kofman *et al*, 1995, 1996). For $\lambda \sim 10^{-13}$ this means than nonthermal perturbations produced at reheating may restore symmetry on the scale up to $\phi_0 \sim 10^{16}$ GeV.

Later $\langle(\delta\phi)^2\rangle$ decreases as $a^{-2}(t)$ because of the expansion of the universe. This leads to the phase transition with symmetry breaking. The homogeneous component $\phi(t)$ at the moment of the phase transition happens to be significantly less than $\sqrt{\langle(\delta\phi)^2\rangle}$ due to its decay in the regime of the narrow parametric resonance after preheating (Kofman *et al*, 1994): $\overline{\phi^2} \propto t^{-7/6} \propto t^{-1/6}\langle(\delta\phi)^2\rangle$; bar means averaging over oscillations.

The mechanism of symmetry restoration described above is very general; in particular, it explains a surprising behavior of oscillations of the scalar field found numerically in the $O(N)$-symmetric model discussed by Boyanovsky *et al* (1995). It is important that during the interval between preheating and the establishing of thermal equilibrium the universe could experience a series of phase transitions which we did not anticipate before. For example, cosmic strings and textures, which could be an additional source for the formation of the large scale structure of the universe, should have $\phi_0 \sim 10^{16}$ GeV (Vilenkin and Shellard, 1994). To produce them by thermal phase transitions in our model one should have the temperature after reheating greater than 10^{16} GeV, which is extremely hard to obtain (Kofman and Linde, 1987). Even with an account taken of the stage of explosive reheating, the resulting reheating temperature typically remains many orders of magnitude smaller than 10^{14} GeV, since it is mainly determined by the last stages of reheating where the parametric resonance is inefficient. Meanwhile, as we see now, fluctuations produced during the first stage of reheating are more than sufficient to restore the symmetry. Then the topological defects can be produced in a standard way when the symmetry breaks down again. In other words, production of superheavy topological defects can be easily compatible with inflation.

On the other hand, the topological defect production can be quite dangerous. For example, the model (8) of a one-component real scalar field ϕ has a discrete symmetry $\phi \to -\phi$. As a result, after the phase transition induced by fluctuations $\langle(\delta\phi)^2\rangle$ the universe may become filled with domain walls separating phases $\phi = +\phi_0$ and $\phi = -\phi_0$. This is expected to lead to a cosmological disaster.

This question requires a more detailed analysis. Even though the point $\phi = 0$ after preheating becomes a minimum of the effective potential, the field ϕ continues oscillating around this minimum. Therefore, at the moment t_c it may happen to be either to the right of the maximum of $V(\phi)$ or to the left of it everywhere in the universe. In this case the symmetry breaking will occur in one preferable direction, and no domain walls will be produced. A similar mechanism may suppress production of other topological defects.

However, this would be correct only if the magnitude of fluctuations $(\delta\phi)^2$ were smaller than the average amplitude of the oscillations $\overline{\phi^2}$. In our case fluctuations $(\delta\phi)^2$ are greater than $\overline{\phi^2}$ (Kofman *et al*, 1994), and they can have considerable local deviations from their average value $\langle(\delta\phi)^2\rangle$. Investigation of this question shows that in the theory (8) with $\phi_0 \ll 10^{16}$ GeV fluctuations destroy the coherent distribution of the oscillating field ϕ and divide the universe into equal number of domains with $\phi = \pm\phi_0$, which leads to the domain wall problem. This means that in consistent inflationary models of the type of (8) one should have either $\phi_0 = 0$ (no symmetry breaking), or $\phi_0 \gtrsim 10^{16}$ GeV.

Now we will consider models where the symmetry breaking occurs for fields other than the inflaton field ϕ. The simplest model has an effective potential

$$V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{\alpha}{4}\Big(\chi^2 - \frac{M^2}{\alpha}\Big)^2 + \frac{1}{2}g^2\phi^2\chi^2 \ . \tag{12}$$

The models of such type have been studied in (Kofman and Linde, 1987, Linde, 1991, 1994).

We will assume here that $\lambda \ll \alpha, g^2$, so that at large ϕ the curvature of the potential in the χ-direction is much greater than in the ϕ-direction. In this case at large ϕ the field χ rapidly rolls toward $\chi = 0$. An interesting feature of such models is the symmetry restoration for the field χ for $\phi > \phi_c = M/g$, and symmetry breaking when the inflaton field ϕ becomes smaller than ϕ_c. As was emphasized in (Kofman and Linde, 1987), such phase transitions may lead to formation of topological defects without any need for high-temperature effects.

We would like to point out some other specific features of such models. If the phase transition discussed above happens during inflation (Kofman and Linde, 1987) (i.e. if $\phi_c > M_p$ in our model), then no new phase transitions occur in this model after reheating. However, for $\phi_c \ll M_p$ the situation is much more complicated. First of all, in this case the field ϕ oscillates with the initial amplitude $\sim M_p$ (if $M^4 < \alpha\lambda M_p^4$). This means that each time when the absolute value of the field becomes smaller than ϕ_c, the phase transition with symmetry breaking occurs and topological defects are produced. Then the absolute value of the oscillating field ϕ again becomes greater than ϕ_c, and symmetry restores again. However, this regime does not continue for a too long time. Within a few dozen oscillations, quantum fluctuations of the field χ will be generated with the dispersion $\langle(\delta\chi)^2\rangle \sim C^2 g^{-1}\sqrt{\lambda} M_{\rm P}^2 \ln^{-2}\frac{1}{g^2}$ (Kofman *et al*, 1995, 1996). For $M^2 < C^2 g^{-1}\sqrt{\lambda}\alpha M_p^2 \ln^{-2}\frac{1}{g^2}$, these fluctuations will keep the symmetry restored. Note that this effect may be even stronger if instead of the term $\frac{\lambda}{4}\phi^4$ we would consider $\frac{m^2}{2}\phi^2$, since in that case the resonance is more broad (Kofman *et al*, 1994). The symmetry breaking finally completes when $\langle(\delta\chi)^2\rangle$ becomes small enough.

One may imagine even more complicated scenario when oscillations of the scalar field ϕ create large fluctuations of the field χ, which in their turn interact with the scalar fields Φ breaking symmetry in GUTs. Then we would have phase transitions in GUTs induced by the fluctuations of the field χ. This means that no longer can the absence of primordial monopoles be considered as an automatic consequence of inflation. To avoid the monopole production one should use the theories where quantum fluctuations produced during preheating are small or decoupled from the GUT sector. This condition imposes additional constraints on realistic inflationary models. On the other hand, preheating may remove some previously existing constraints on inflationary theory. For example, in the models of GUT baryogenesis it was assumed that the GUT symmetry was restored by high temperature effects, since otherwise the density of X, Y, and superheavy Higgs bosons would be very small. This condition is hardly compatible with inflation. It was also required that the products of decay of these particles should stay out of thermal equilibrium, which is a very restrictive condition. In our case the superheavy particles responsible for baryogenesis can be abundantly produced by parametric resonance, and the products of their decay will not be in a state of thermal equilibrium until the end of reheating.

Now let us return to the theory (fp1) including the field χ for $g^2 \gg \lambda$. In this case the main fraction of the potential energy density $\sim \lambda M_{\rm P}^4$ of the field ϕ predominantly transfers to the energy of fluctuations of the field χ due to the explosive χ-particles creation in the broad parametric resonance. The dispersion of fluctuations after preheating is $\langle(\delta\chi)^2\rangle \sim C^2 g^{-1}\sqrt{\lambda} M_{\rm P}^2 \ln^{-2}\frac{1}{g^2}$. These fluctuations lead to the symmetry restoration in the theory (fp1) with $\phi_0 \ll C\left(\frac{g^2}{\lambda}\right)^{1/4} M_p \ln^{-1}\frac{1}{g^2}$,

which may be much greater than 10^{16} GeV for $g^2 \gg \lambda$.

Later the process of decay of the field ϕ continues, but, just as in the model described in the previous section, one may say with a good accuracy that the fluctuations $\langle(\delta\chi)^2\rangle$ decrease as $g^{-1}\sqrt{\lambda}M_p^2\,(\frac{a_i}{a(t)})^2$ and their energy density ρ decreases as the energy density of ultrarelativistic matter, $\rho(t) \sim \lambda M_p^4\,(\frac{a_i}{a(t)})^4$, where a_i is the scale factor at the end of inflation. This energy density becomes equal to the vacuum energy density $\frac{m^4}{4\lambda}$ at $a_0 \sim a_i\,\sqrt{\lambda}M_p/m$, $t \sim \sqrt{\lambda}M_p m^{-2}$. Since that time and until the time of the phase transition with symmetry breaking the vacuum energy dominates, and the universe enters secondary stage of inflation.

The phase transition with spontaneous symmetry breaking occurs when $m_{\phi,eff} = 0$, $\langle(\delta\chi)^2\rangle = g^{-2}m^2$. This happens at $a_c = a_i\,\lambda^{1/4}g^{1/2}M_p/m$. Thus, during this additional period of inflation the universe expands $\frac{a_c}{a_0} \sim \sqrt{g}\,\sqrt{\phi_0/m} = (g^2/\lambda)^{1/4}$ times. This is greater than expansion during thermal inflation (fp5a) by the factor $O(g^{-1/2})$, and in our case inflation occurs even if $g^4 \ll \lambda$.

In this example we considered the second stage of inflation driven by the inflaton field ϕ. However, the same effect can occur in theories where other scalar fields are coupled to the field χ. For example, in the theories of the type of (fp5b) fluctuations $\langle(\delta\chi)^2\rangle$ produced at the first stage of reheating by the oscillating inflaton field ϕ lead to a secondary inflation driven by the potential energy of the "flaton" field Φ. During this stage the universe expands $\sim \sqrt{g}\,\sqrt{\Phi_0/m}$ times. To have a long enough inflation one may consider, e.g., supersymmetric theories with $m \sim 10^2$ GeV and $\Phi_0 \sim 10^{12}$ (Lyth and Stewart, 1995). This gives a relatively long stage of inflation with $\frac{a_c}{a_0} \sim \sqrt{g}\,10^5$, which may be enough to solve the Polonyi field problem if the constant g is not too small.

If the coupling constant g is sufficiently large, fluctuations of the field χ will thermalize during this inflationary stage. Then the end of this stage will be determined by the standard theory of high temperature phase transition, and the degree of expansion during this stage will be given by $10^{-1}g\,\sqrt{\Phi_0/m}$, see eq. (fp5a). It is important, however, that the inflationary stage may begin even if the field χ has not been thermalized at that time.

The stage of inflation described above occurs in the theory with a potential which is not particularly flat near the origin. But what happens in the models which have flat potentials, like the original new inflation model in the Coleman-Weinberg theory? One of the main problems of inflation in such models was to understand why should the scalar field ϕ jump onto the top of its effective potential, since this field in realistic inflationary model is extremely weakly interacting and, therefore, it could not be in the state of thermal equilibrium in the very early universe. Thus, it is much more natural for inflation in the Coleman-Weinberg theory to begin at very large ϕ, as in the simplest version of chaotic inflation in the theory $\lambda\phi^4$. However, during the first few oscillations of the scalar field ϕ at the end of inflation in this model, it produces large non-thermal perturbations of vector fields $\langle(\delta A_\mu)^2\rangle \sim C^2 g^{-1}\sqrt{\lambda}M_{\rm P}^2 \ln^{-2}\frac{1}{g^2}$. This leads to symmetry restoration and initiates the second stage of inflation beginning at $\phi = 0$. It suggests that in many models inflation most naturally begins at large ϕ as in the simplest version of the chaotic inflation scenario. But then, after the stage of preheating, the second stage of inflation may begin

like in the new inflationary scenario. Thus, the non-thermal symmetry restoration after chaotic inflation may produce initial conditions which are necessary for new inflation.

3 Discussion

Development of inflationary cosmology demonstrates over and over again that it is dangerous to be dogmatic. For many years we believed that if observers find that $\Omega = 1$, they will prove inflation, and they will kill inflation if they find that Ω differs from 1 by more than about 10^{-4}. This made inflation an easy and popular target for observers. Now we have found that there exist several rather simple models of an open inflationary universe, according to which our universe consists of infinitely many domains with all possible values of Ω. This result is very encouraging for theorists and somewhat disappointing for observers. Indeed, at the first glance the measurement of Ω looses its fundamental importance, and inflation becomes a theory which is very difficult to verify. My opinion is quite opposite: we have a win-win situation. If we find that $\Omega = 1$, it will prove inflationary cosmology since 99% of inflationary models predict $\Omega = 1$, and no other theory makes this prediction. On the other hand, if we find that $\Omega \neq 1$, it will not disprove inflation, since now we have inflationary models with $\Omega \neq 1$, and no other models of *homogeneous and isotropic* universe with $\Omega \neq 1$ are known to us so far. Thus, inflationary theory becomes as robust as the whole Big Bang theory, and it has a very nice property: It is possible to prove inflation, and it is very hard to kill it.

On the other hand, until now we believed that inflation automatically solves the primordial monopole problem. We thought that the physical processes after inflation can be well understood as soon as we calculate the value of reheating temperature. We have found that the situation is much more complicated, and, consequently, much more interesting. In addition to the standard high temperature phase transition, there exists a new class of phase transitions which may occur at the intermediate stage between the end of inflation and the establishing of thermal equilibrium. These phase transitions may take place even if the scale of symmetry breaking is very large and the reheating temperature is very small. An important feature of these new phase transitions is their non-universality. Indeed, they occur out of the state of thermal equilibrium. Large quantum fluctuations are generated only for some bose fields interacting with the inflaton field. As a result, it becomes possible to have phase transitions producing superheavy strings, but to avoid the phase transitions producing monopoles. These phase transitions may lead to an efficient GUT baryogenesis, and to existence of a secondary stage of inflation after reheating. Therefore, phase transitions of the new type may have dramatic consequences for inflationary models and the theory of physical processes in the very early universe.

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